

B. Sc. Part I  
Physics Honors

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Relativity

# Galilean Transformation in vector form

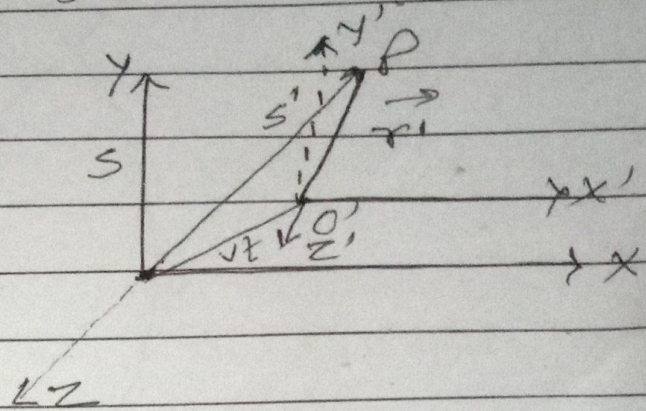
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Consider two systems  $S$  and  $S'$  moving with vector  $\vec{v}$  to  $S$ . Initially origins of two systems coincide.

Let  $\vec{r}$  and  $\vec{r}'$  be the position vectors of any particle (event)  $P$  relative to origins  $O$  and  $O'$  of system  $S$  and  $S'$  respectively after time  $t$ .



Then from fig. using the law of triangle of vector addition

$$\vec{r} = \vec{r}' + \vec{v}t$$

or  $\vec{r}' = \vec{r} - \vec{v}t$  — (3)

also  $t' = t$  — (4)

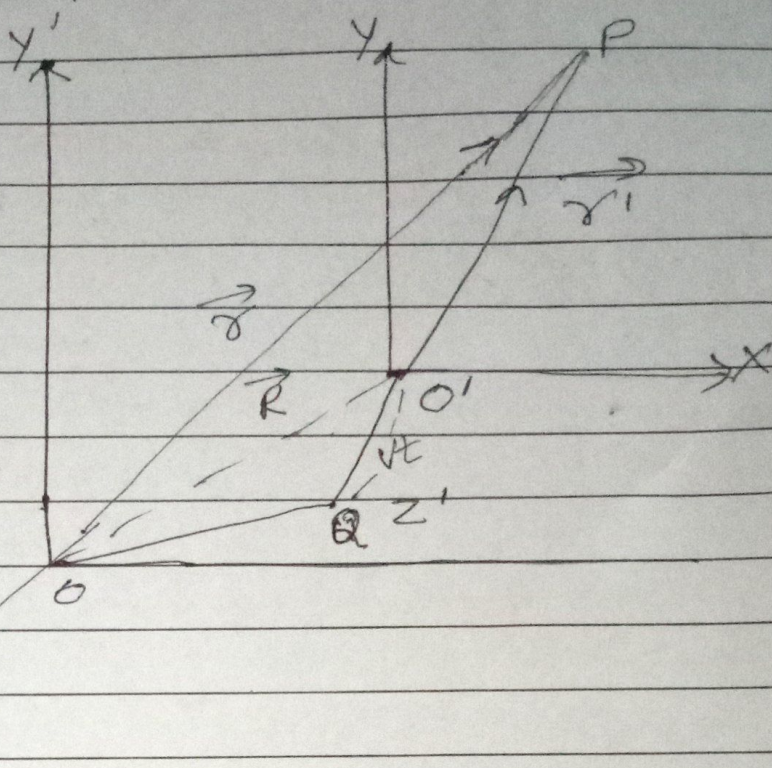
These eqn's represent Galilean transformations of space and time in vector form.

eqn (1), (2) & (3) are called time-dependent Galilean transformations, since they are time dependent and were obtained by Galileo.

## Problem - 1

Prove that the Galilean transformation of a position vector is expressed as  $\vec{r} = \vec{r}_0 + \vec{r}' + \vec{v}t$ , where  $\vec{v}$  is the linear velocity of the frame  $O'$  and  $\vec{r}_0$  is the position vector of origin  $O'$  as measured by  $O$  at  $t' = 0$ .

Consider two frames  $S$  &  $S'$  The latter moving with velocity  $\vec{v}$  relative to forms. Let  $O$  and  $O'$  be the observers situated in  $S$  and  $S'$  respectively observing the event happening at  $P$ .



If  $\vec{r}$  and  $\vec{r}'$  are the position vector of the point  $P$  at any constant.

Then we have from fig

$$\vec{r} = \vec{r}' + \vec{R} \quad \text{--- (1)}$$

Where  $\vec{R}$  is the position to  $O$  after time  $t$ .

If  $\vec{r}_0 = \vec{OQ}$  is the position vector of the observer  $O'$  relative to  $O$  at  $t=0$

Then from above fig

$$\vec{R} = \vec{OQ} + \vec{QO'} = \vec{r}_0 + \vec{vt} \quad \text{--- (2)}$$

Since the distance traversed ( $\vec{QO'}$ ) by the observer  $O'$  in time  $t$  is  $\vec{vt}$  where  $\vec{v}$  is the velocity of the  $O'$  relative to  $O$ .

Putting the value of  $\vec{R}$  from (2) in (1)

we get 
$$\vec{r} = \vec{r}' + \vec{r}_0 + \vec{vt}$$

$$\vec{r} = \vec{r}_0 + \vec{r}' + \vec{vt}$$

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$$\vec{r} = \vec{r}' + \vec{r}_0 + \vec{v}t$$

$$\vec{r} = \vec{r}_0 + \vec{r}' + \vec{v}t' \quad \text{--- (3)}$$

~~Prob 2~~

Problem 2 - Consider two systems  $S$  &  $S'$ ,  $S'$  moving with velocity  $\vec{v} = iV_x + jV_y + kV_z$  relative to  $S$ . At the origins of the two systems coincide at  $t = t' = t_0$  find the Galilean transformation equations.

Soln. The system  $S'$  is moving relative to  $S$  with velocities  $V_x, V_y$  &  $V_z$  along +ve directions of  $x, y$  and  $z$  axes respectively.

If the origins of two frames coincide at

$$t = t' = t_0 \quad \text{Then}$$

The distance traversed by observer  $O'$  in  $S'$  relative to observer  $O$  in  $S$  at any instant  $t$  along axis of

$$x = V_x (t - t_0)$$

The distance traversed by  $O'$  relative to  $O$  at any instant  $t$  along axis of  $y = V_y (t - t_0)$

The distance traversed by  $O'$  relative to  $O$  at any instant

$$\text{along } z \text{ axis} = V_z (t - t_0)$$

Thus the Galilean transformation equations are given by

$$x' = x - V_x (t - t_0)$$

$$y' = y - V_y (t - t_0)$$

$$z' = z - V_z (t - t_0)$$

Proved